Greek Letters

α = drag-flow constant
β = pressure-flow constant
ϕ = helix angle
λ = distance measured along screw channel, in.
μ = viscosity, lb./in.-min.
θ = residence time, min.

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A Phenomenological Interpretation and Correlation of Drag Reduction

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Drag reduction data for five concentrations of a water-soluble polymer and three tube diameters are presented. The data are correlated by a single curve relating two dimensionless parameters; these have been obtained from a phenomenological analysis of the mechanism of drag reduction.

Drag reduction, which is observed in turbulent flow of polymer solutions, has recently attracted much attention, and both data and tentative interpretations are fairly abundant in the literature. Nonetheless, no correlation of predictive value and reasonably simple form has so far been proposed. This paper reports the results of a work done in the attempt to establish such a correlation, on a strictly phenomenological basis.

THEORY

It is well known that, if pressure drop data for drag-reducing solutions are plotted in dimensionless form, that is, as friction factor vs. Reynolds number, different curves are observed for different tube diameters. The larger the tube diameter, the smaller is the drag reduction effect. It is therefore necessary, in order to correlate the data, to introduce, in addition to the Reynolds number, a second dimensionless group. This group should contain the tube diameter, in order to be able to correlate the observed diameter effect; therefore, this dimensionless group cannot simply be a dimensionless rheological parameter of the solution considered.

Astarita (*1*) proposed an interpretation of the drag reduction phenomenon based on the comparison of the smallest frequency of the noninviscid (or energy-dissipating) eddies, ν, and some characteristic time scale of the fluid considered, T. Following arguments of Levich (*2*) and of Lumley (*3*), the relevant frequency ν was evaluated as:

$$\nu = \frac{U}{D} N_{Re}^{0.5} \tag{1}$$

The proper dimensionless group to be considered, following this line of thought, is given by the product νT. This group can be identified with the Deborah number of the flow, as discussed in some recent papers (*4* to *6*).

Seyer and Metzner (*6*) have re-elaborated Astarita's discussion, and, following arguments of Hinze (*7*), propose to calculate ν by the equation:

$$\nu = \frac{U}{D} N_{Re}^{0.75} \tag{2}$$

Equation (2) seems to be preferable as compared to Equation (1) on the basis of the following considerations. It has often been reported (*8*, *9*) that drag reduction is observed only at wall shear stresses τ_w exceeding some critical value, τ_{cr}. This can also be stated in reverse form, say, that no drag reduction is observed if:

$$\tau_w < \tau_{cr} \tag{3}$$

Therefore, as long as Equation (3) is satisfied, τ_w can be calculated through the usual friction factors correlations, say by means of Blasius equation:

$$\tau_w = \frac{f}{2} \rho U^2 = 0.0395 \ \rho U^2 \ N_{Re}^{-0.25} \tag{4}$$

Equation (3) can be rearranged as follows:

$$\tau_w = 0.0395 \ \mu_w \frac{U}{D} \ N_{Re}^{0.75} \tag{5}$$

where μ_w is the viscosity at the tube wall. Equation (5) is based on the simplifying hypothesis that, for non-Newtonian drag-reducing solutions, the Reynolds number is defined on the basis of the wall viscosity, and that Equation (4) is still valid with this definition when no drag reduction occurs. This is of course only an approximation, as shown by Dodge and Metzner's correlation (*10*) of friction factors for nondrag reducing non-Newtonian liquids, but presumably a quite satisfactory one for slightly non-Newtonian liquids, as most drag-reducing dilute polymer solutions are.

If Equation (5) is substituted into Equation (3), the condition for the onset of drag reduction is obtained as

$$\frac{U}{D} N_{Re}^{0.75} < 25.3 \frac{\tau_{cr}}{\mu_w} \tag{6}$$

which is consistent with the preceding arguments provided that ν is calculated from Equation (2) and 0.0395 μ_w/τ_{cr} is regarded as proportional to the characteristic time scale of the liquid. Incidentally, this appears to be an interesting possibility for estimating the magnitude of the time scale T. For dilute polymer solutions, values of τ_{cr} of the order of 50 dynes/sq.cm. have been reported (*8*); this would correspond, with a wall viscosity of the order of 2 centipoise, to a limiting frequency of about 6×10^4 sec.$^{-1}$.

Dimensional considerations suggest that the friction factor f, should be a function of both the Reynolds number and the Deborah number:

$$f = f(N_{Re}, \nu T) \tag{7}$$

It would be particularly useful to somehow break Equation (7) into the separate influences of the Reynolds and Deborah numbers. This was attempted by Seyer and Metzner (*6*), who define a fractional drag reduction, F, as:

$$F = \frac{f^0 - f}{f^0 - f_L} \tag{8}$$

where f^0 is the friction factor which would be observed at the same Reynolds number in the absence of drag reduction, and f_L is the friction factor calculated from Poiseuille's equation:

$$f_L = \frac{16}{N_{Re}} \tag{9}$$

Equation (8) is based on the consideration that the friction factor cannot be less than would be observed if laminar flow were stabilized at the Reynolds number of interest, say $f \geqq f_L$. The quantity F is thus bound to values between 0 (no drag reduction) and 1; the latter value corresponds to a complete drag reduction, with the pressure drop given by Poiseuille's equation.

Unfortunately, the quantity F for Seyer and Metzner's data turns out not to be a unique function of the Deborah number, but still depends on both N_{Re} and N_{De}. There is therefore no advantage obtained by introducing the quantity F. The residual dependency of F on N_{Re} in Seyer and Metzner's data may be due to the highly non-Newtonian character of the solutions used; this may invalidate Equation (2), which has been obtained for Newtonian liquids.

In this work, we choose to define a drag reduction ratio β as

$$\beta = f/f^0 \tag{10}$$

which, as shown below, is more powerful in eliminating the residual dependency on Reynolds number. The quantities F and β are related to each other through the following equation:

$$F = \frac{1 - \beta}{1 - f_L/f^0} \tag{11}$$

At very large Reynolds numbers, f_L/f^0 tends to zero, and thus $F \simeq 1 - \beta$. The quantity β is bound between the values 1 (no drag reduction) and f_L/f^0.

Let us assume that the value of β is uniquely determined by the Deborah number of the flow:

$$\beta = \beta\ (\nu T) \tag{12}$$

The function $\beta(\ )$ is expected to be monotonically decreasing; this behavior at least qualitatively justifies the diameter effect observed.

For any given drag-reducing liquid, the value of T is a constant, and thus β should be a unique function of ν. Thus, a first check of the validity of Equation (12) can be obtained by plotting β vs. ν; in such a plot no residual diameter effect should be observed.

If different solutions need to be compared with each other, the value of T should be known. This presents two difficulties. The first one is that T is not unequivocally defined in terms of measurable rheological properties (*11*, *12*); Seyer and Metzner choose a definition of T which yields a nonconstant value, that is, T turns out to be a function of shear rate. A problem then arises as to the proper value of T to use. The second difficulty is that rheological measurements needed to evaluate T are in practice impossible at very low polymer concentration [though Oliver (*13*) has reported some data of interest in this regard]. While Seyer and Metzner have chosen to overcome this difficulty by working with concentrated solutions, it should be observed that dilute solutions are of greater pragmatic interest in drag reduction studies, and moreover they are approximately Newtonian, and thus the problems of the internal consistency of Equation (4) and (5) and of the appropriate definition of the Reynolds number are only minor.

Fortunately enough, the arguments developed so far can be applied without actual knowledge of the value of T. In fact, let $\nu_{0.5}$ be the frequency corresponding to a drag reduction ratio of 0.5:

$$0.5 = \beta\ (\nu_{0.5} T) = \beta\ (K) \tag{13}$$

where $K = \nu_{0.5} T$ is a constant which, if Equation (12) is valid, does not depend on the particular solution considered.

Equation (12) can now be written in the equivalent form:

$$\beta = \beta \left(\frac{\nu K}{\nu_{0.5}} \right) = \beta' \left(\frac{\nu}{\nu_{0.5}} \right) \tag{14}$$

where the measurable characteristic parameter $\nu_{0.5}$ has been substituted for the troublesome parameter T. Of course, knowledge of the value of K (and thus of the relationship between T and $\nu_{0.5}$) would require a deep understanding of the mechanism of drag reduction; but from a phenomenological viewpoint, Equation (14) is equivalent to Equation (12), and therefore the behaviors of different solutions can be compared with each other without knowing the value of T. In fact, if Equation (12) is valid, drag reduction data for different solutions and different diameters should be correlated by a single curve if plotted in the form of β vs. $\nu/\nu_{0.5}$.

Of course, such a correlation has predictive value only if, for any given solution, enough pressure drop data are taken as required to evaluate $\nu_{0.5}$. This is an unescapable drawback, as long as there are no independent ways of evaluating T. An attempt at the solution of the latter problem can be found in recent work correlating drag reduction to molecular parameters of the polymeric solute, such as reported for example by Virk, et al. (*14*).

EXPERIMENTAL PROCEDURE

A flow loop consisting of reservoir tank, positive displacement pump, by-pass with flow rate control valves, test sections and flow rate measurement unit was used for the experiments. Three test sections could be assembled simultaneously in parallel, with independent control valves. The test sections were different lengths of lucite tubing, of three different diameters, 0.69, 1.10, and 2.05 cm. The length of the test section was, for most of the experiments, 250 cm. The highest flowrates through the smallest diameter tube were obtained with a shorter tube (50 cm.), because an upper limit to the total pressure drop was set by the pump's maximum head. The short tube was equipped with a conical inlet section and a Venturi exit section in order to minimize concentrated losses.

Pressure taps were mounted flush to the tube wall; at any measuring point four holes were drilled in the tube, and an outer shell obtained from a larger diameter tube was thus

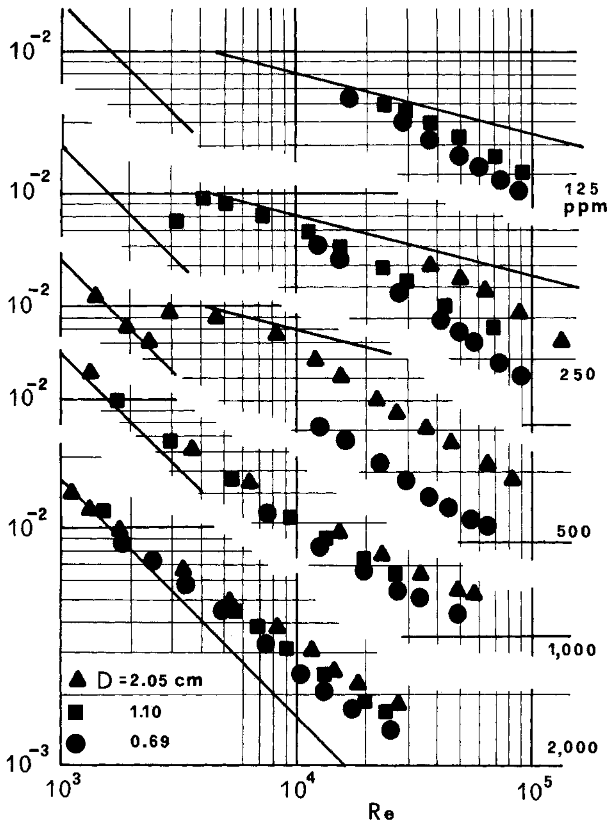

Fig. 1. Friction factor sample data. The f_L lines (left) and the f^0 lines (up) are reported for comparison.

brought at the average pressure in the section. At least 10 measuring points were provided for each tube; the pressure differences could be read either on a test solution–carbon tetrachloride or on a test solution–mercury differential manometers.

Shear stress-shear rate data were taken for each solution before and after each series of runs by means of a conventional capillary viscometer. When the reproducibility was poor, the series was discarded from further consideration.

The solution used were aqueous solutions of ET597, a commercial water-soluble additive with concentrations ranging from 125 to 2,000 ppm. Shear stress-shear rate curves were almost linear at the lowest concentration, and even for the 1,000 ppm. solution the flow index was 0.83, that is, the solution was approximately Newtonian. At 2,000 ppm., a curved shear stress-shear rate line was observed, with flow behavior index as low as 0.56 at the lowest shear rates.

The data were elaborated to friction factor-Reynolds number data making use of the definition of Dodge and Metzner (*10*) for the Reynolds number. Though this is not exactly consistent with the argument leading to Equation (5), this definition of the Reynolds number was preferred because on this basis laminar flow data are represented by the usual Newtonian Equation, $f = 16/N_{Re}$. As will be shown, this choice seems inadequate for the 2,000 ppm. solution, which is highly non-Newtonian.

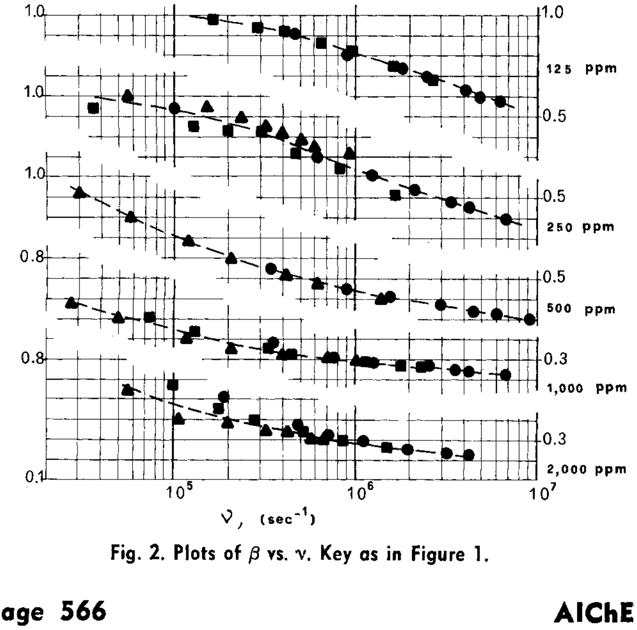

Fig. 2. Plots of β vs. ν. Key as in Figure 1.

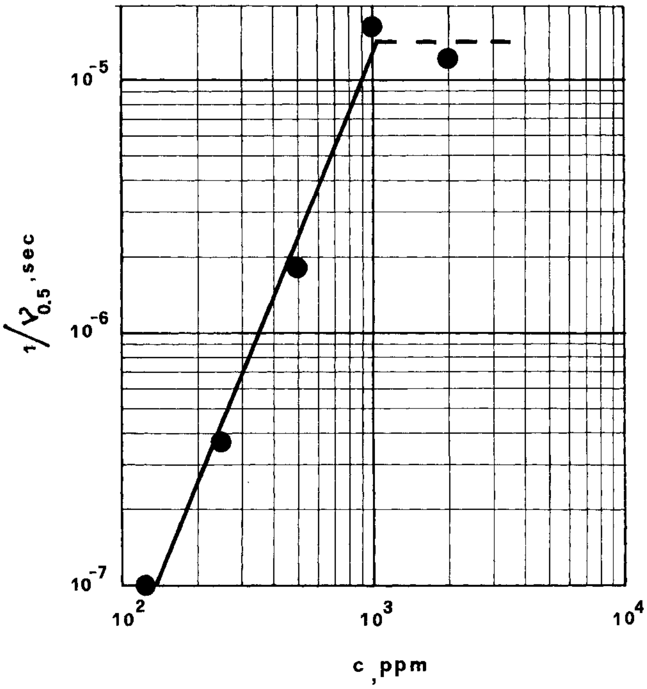

Fig. 3. Dependency of $\nu_{0.5}$ on polymer concentration.

Original data are available (*15*); only some representative data are reported in the following. The range of independent variables covered was polymer concentration, 125 to 2,000 ppm.; tube diameter, 0.69 to 2.05 cm.; Reynolds number, 2×10^3 to 2×10^5.

RESULTS AND DISCUSSION

Figure 1 reports representative data for those series of runs which were not discarded because of experimental difficulties. Each group of data is relative to a single solution; the two or three curves within each group are relative to different tube diameters. The typical diameter effect is observed, say the smallest diameter tube always gives the smallest values for the friction factor.

Sample data were elaborated to the form suggested by Equation (12), and are plotted in Figure 2 as β vs. ν. The diameter effect seems to have been adequately taken into account; in fact, no systematic shift is observed. Only for the 250 ppm. solution a diameter effect still appears to exist, but the lowest data refer to the middle diameter.

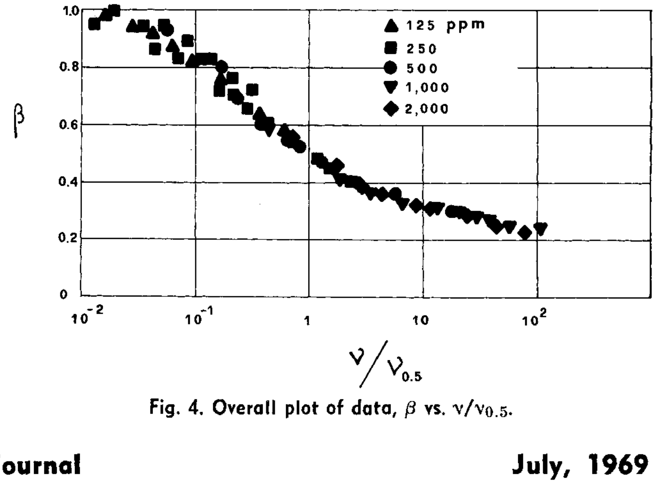

Fig. 4. Overall plot of data, β vs. $\nu/\nu_{0.5}$.

This seems to imply that the observed shift is not significant.

Values of $\nu_{0.5}$ have been obtained from the smooth curves through the data in Figure 2. These are plotted in Figure 3 as $T/K = 1/\nu_{0.5}$ vs. polymer concentration c. A straight line appears to correlate the data reasonably well up to a concentration of 1,000 ppm., suggesting that for dilute solutions

$$T \propto c^{2.5} \tag{15}$$

At higher concentrations, T seems to be approximately independent of concentration. The exponent 2.5 in Equation (15) compares favorably with the value 2.0 resulting from existing molecular theories; in fact, the natural time being a measure of second-order effects resulting from polymer-polymer interactions, a dependency on the square of concentration would be predicted from the simplest approach (*16*).

Finally, the data reported in Figure 2 have been plotted in the form suggested by Equation (14) as β vs. $\nu/\nu_{0.5}$, see Figure 4. A single curve correlates all the data obtained, over a range of values of the independent variable $\nu/\nu_{0.5}$ covering almost six orders of magnitude. The scattering of the data about the curve does not exceed 5%, which is to be considered excellent in view of the very wide range of variable covered. The only limitation is that solutions of only one type of polymer have been tested, and therefore the possibility that the curve β' $(\nu/\nu_{0.5})$ has a different shape for different polymers cannot be excluded.

COMPARISON WITH PREVIOUS RESULTS

Though published drag-reduction data are relatively abundant in the literature, it is generally difficult to compare such data with the correlation proposed in the present work. Often rheological data are not reported, so that the value of ν cannot be calculated from the reported friction factor-Reynolds number plots; moreover, reported data often correspond either to β values always larger than 0.5, or always smaller.

Seyer and Metzner's data are the most useful ones in this regard, though again most β values are smaller than 0.5. As discussed above, these data are elaborated to F values, which turn out to depend both on νT and on the Reynolds number. This is in contrast with the present results, and two possible explanations for this discrepancy are offered here.

The first reason may lie in the way Seyer and Metzner evaluate T, which is not unequivocal. The Deborah number is calculated by taking T as the natural time corresponding to the wall shear rate, a choice which may not be the wisest one. It is possible that the shift of the $F(\nu T)$ curves with Reynolds number would disappear if $1/\nu_{0.5}$ were used instead of T; this check is unfortunately impossible because, for most of Seyer and Metzner's data, $\beta <$ 0.5. The second reason for the discrepancy may be the highly non-Newtonian character of the solutions used by Seyer and Metzner, which may invalidate the argument leading to Equation (2).

Seyer and Metzner have observed a very minor influence of polymer concentration on both the $f/2 - N_{Re}$ curves, and on the value of T. This is not at variance with the present results. In fact, Figure 1 and Figure 3 show that, at polymer concentrations exceeding 1,000 ppm., the dependency of drag reduction on polymer concentration levels out. Seyer and Metzner used the same polymer as used in the present work, but their range of concentration was between 2,000 and 6,000 ppm.

Figure 3 shows that the value of $1/\nu_{0.5}$ at high polymer concentrations is of the order of 10^{-5} sec. T values reported by Seyer and Metzner range between 8×10^{-3} sec. (at a shear rate of 10^3 sec.$^{-1}$) to 4×10^{-4} sec. (at a shear rate of 4×10^4 sec.$^{-1}$). The concentration dependency of T decreases with increasing shear rate. It is thus possible that the relevant value of T should be much less than the reported ones, say corresponding to a very large shear rate, that is, not the wall shear rate, but the one prevailing on the microscale of the turbulent core. The arguments leading to Equations (1) or (2) are indeed relative to the smallest turbulent eddies. If this line of thought is correct, the value of T would be in reasonable agreement with the values of $1/\nu_{0.5}$ observed in the present work at concentrations exceeding 1,000 ppm.

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NOTATION

D = tube diameter, cm.
f = friction factor
f^0 = friction factor without drag reduction
f_L = laminar flow friction factor
F = fractional drag reduction, see Equation (8)
K = $\nu_{0.5}T$
N_{De} = Deborahs number
N_{Re} = Reynolds number
T = characteristic time, sec.
U = average velocity, cm./sec.
β = drag reduction ratio, f/f^0
μ_w = viscosity at the wall, poise
ν = characteristic frequency, sec.$^{-1}$
$\nu_{0.5}$ = value of ν for which $\beta = 0.5$
ρ = density, g.-cm.$^{-3}$
τ_{cr} = minimum wall shear stress for drag reduction, dynes/sq.cm.
τ_w = wall shear stress, dynes/sq.cm.

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